Maths with Spirograph

Advanced Version

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Stimulate problem solving	**Gold**	**Stimulate entrepreneurship**	**Silver**
Stimulate creativity	**Gold**	**Informal learning enviro.**	**Platinum**
Stimulate critical thinking	**Gold**	**Technology use**	**Platinum**
Stimulate group work	**Gold**		

Practicalities

Preparation: 15min

Duration: 3hrs

Material needs:

- Pencils and paper sheets
- Plexiglass sheets or cardboard
- Laser cutter
- Or 3D-printer
- PCs or laptops

Group size range: 25-30

Ideal sub-group size: 3-4

Workshop made for: 12-16 years old
Easily transferable to workshops for ages between: -12/+16

Environment FabLab necessary: Yes

Educational area:
* Mathematics
* Technology
* Visual Arts

Precognition

Knowledge about arithmetic and integers. Understanding of definition of Least Common Multiple of two or more integer numbers and how to find it.

(see box ‘content links’ below)

Preparation

Teacher familiarize herself/himself with the operation of laser-cutter machine (preferably) or with 3D-printer that will be utilized during the workshop.

Splits and arranges students in small groups of 3-4 persons.

Distributes pencils or pens of various colors and paper sheets to each group.

Workshop Guidelines

Phase 1: Orientation and Instruction Phase

Material needs: pencils and paper sheets

Optional: textbook of Maths or access to online educational resources

Goals:

Skill Goals (**Blue**)

(S1) Optional To look up for information independently

(S2) To be able to do basic arithmetic operations

Content Goals (**Green**)

(C1) Understand and acquire content knowledge of subjects of integers and least common multiple

Background story:

We are in charge of a charity group that would like to distribute 150 notebooks, 90 pencils and 60 erasers in promotion packets during a fundraising campaign e.g. about climate change. How many identical packets can we make so that all contain the same number of items from the three categories?

After the completion of these tasks/activities the teacher explains students what a spirograph is and that they are going to make a real one using modern machinery. At this point she/he does not reveal how or why this is related to the maths subject they just worked on.

Goals	Activities	Duration
S1	Ask students to look up for information about the definition of least common multiple of integer numbers and how it is calculated.	To be chosen
S2	Ask students to try to solve the teaser problem above or something similar	10min
C1	Let students to experiment with various similar problems with two or more integer numbers they pick up themselves.	Max: 10’

Phase 2: Design Phase

Students are given ready-to-make drawings of spirographs for reference (see attached at the end of this document) to directly laser cut or 3D-print and familiarize themselves with them. Then they may use drawing software (e.g. INKSCAPE which is free to download and use from https://inkscape.org/) to design their own spirographs or make variations to the basic one. (see attached screenshot of basic steps to follow at the end of this document)

Material needs: PCs or laptops to download and install Inkscape drawing software or equivalent. Plexiglass sheets or cardboard for laser cutter. Instead of laser cutter, a 3D-printer can be used

Goals:

Skill Goals (**Blue**)

(S1) Abstraction of a mathematical concept to a tangible object or model

(S2) Digital literacy, use of ICT for drawing and design

Content Goals (**Green**)

(C1) Experiment, identify and understand the strengths and weaknesses of different materials

(C2) Operating modern manufacturing/prototyping equipment such as laser cutter or/and 3D printer

Goals	Activities	Duration
S1, S2, C1, C2	Student teams design and build initial prototype versions of given spirograph designs with different materials e.g. plexiglass, paperboard, wood sheets etc. They try to understand the properties of each material and what is best for. By doing so they try to devise an optimal design, change and adapt it to achieve their goal.	40 min

Phase 3: Making Phase

Material needs: Plexiglass sheets or cardboard for laser cutter. Instead of laser cutter, a 3D-printer can be used

Goals:

Skill Goals (**Blue**)

(S1) Working in group

(S2) Working under constraints (time pressure, safety regulations)

Content Goals (**Green**)

(C1) Experiment, identify and understand the strengths and weaknesses of different materials

Goals	Activities	Duration
S1, S2, C1	Student teams use laser cutter to make the given spirograph designs with different materials e.g. plexiglass, paperboard, wood sheets etc. They follow mandatory precaution and safety measures and instructions for the operation of equipment	20 min

Phase 4: Operational Phase

Material needs:

Essential: *Spirographs made in previous phase. Pencils or pens of various colors*

Goals:

Skill Goals (**Blue**)

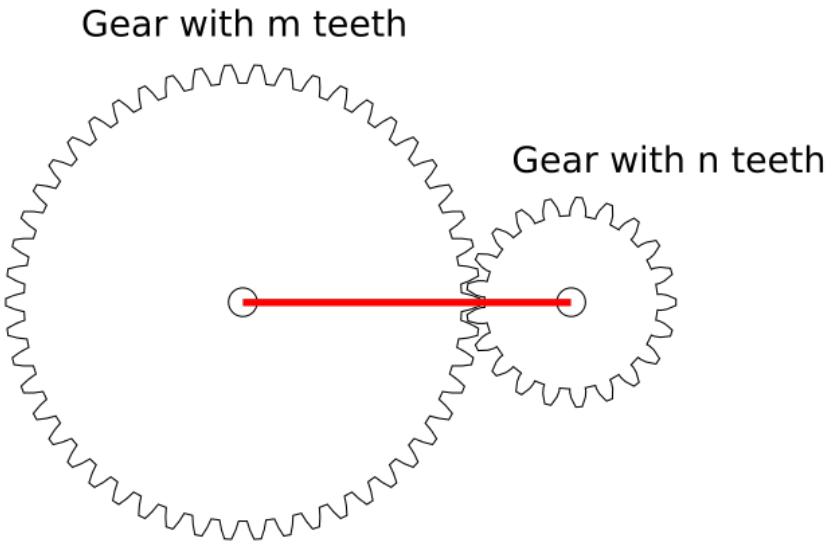
(S1) Trial and error / Deal with failure

(S2) Abstraction of a mathematical concept to a tangible object or model

Content Goals (**Green**)

(C1) Practical understanding and acquisition of content knowledge of mathematical subjects of integers and least common multiple

Goals	Activities	Duration
S1, S2, C1	In the first subphase students are instructed to carefully count and note down the number of teeth of the gears of their spirograph at hand. They use pencils or pens of various colors to draw beautiful spirograph curves.	10 min
S1, S2, C1	They then asked if they can predict after how many revolutions the curves start again for a given set of two gears (see gears problem and drawing below). Is this related to least common multiple? They try and test their predictions. *Gears problem* *Suppose there are two meshing gears in a machine, having m and n teeth, respectively, and the gears are marked by a line segment drawn from the center of the first gear to the center of the second gear. When the gears begin rotating, we can determine how many rotations the first gear must complete to realign the line segment by making use of least common multiple of m and n integers, usually denoted by LCM(m, n). The first gear must complete LCM(m, n)/m rotations for the realignment. By that time, the second gear will have made LCM(m, n)/n rotations.*	10 min


Gear with m teeth
Gear with n teeth

Phase 5: Conclusion Phase

Material needs: *Multicolor spirograph drawings made in previous phase*

Goals:

Skill Goals (**Blue**)
(S1) Abstraction of a mathematical concept to a tangible object or model
(S2) Visualization and demonstration of a mathematical concept

Content Goals (**Green**)
(C1) Clear understanding and acquisition of content knowledge of mathematical subjects of integers and least common multiple
(C2) Create link between geometrical patterns and visual arts

Goals	Activities	Duration
S1, S2, C1, C2	Each team selects its best multicolor spirograph drawing that demonstrates clearly the notion of least common multiple of two integer numbers. Students should be able to explain verbally their conclusions (see Gears problem in previous phase). Also students may be asked to reflect on the activity, for example talking about some difficulties they had, how they dealt with these problems etc.	10 min

Pedagogical tips

The workshop can be conducted in different order of phases (e.g. start with making and playing with a spirograph and then introduce the mathematical concept of least common multiple to study on) depending on the level of students. In this way students may link more easily the abstract/mathematical and tangible aspects of the learning activities in the workshop.

How to transfer to (non-)Fablab environment

Transfer to non-fablab environment is feasible. In one case the necessary building materials are sheets of easy to cut foamboards or paperboard. Instead of gears students can draw and cut circles to make a spirograph. Close supervision and care are needed when students use hobby knifes or cutters to cut their drawings.

Alternatively, simple plastic or paper spirographs can be bought at low cost from arts and crafts or hobby shops, gadget and fun shops etc.

Evaluation of achievements

At the end of the workshop teacher or a committee of volunteer students can award different teams or students individually depending on achievements.
For example, award for:

- The most aesthetically pleasing multicolor spirograph drawing
- The most well-built spirograph
- The most quiet and efficient team of students completing all tasks
- ...

Content links

The workshop can be enhanced with various online educational resources, interactive simulations and online generators which may be available in various languages

Least Common Multiple (Wikipedia): https://en.wikipedia.org/wiki/Least_common_multiple

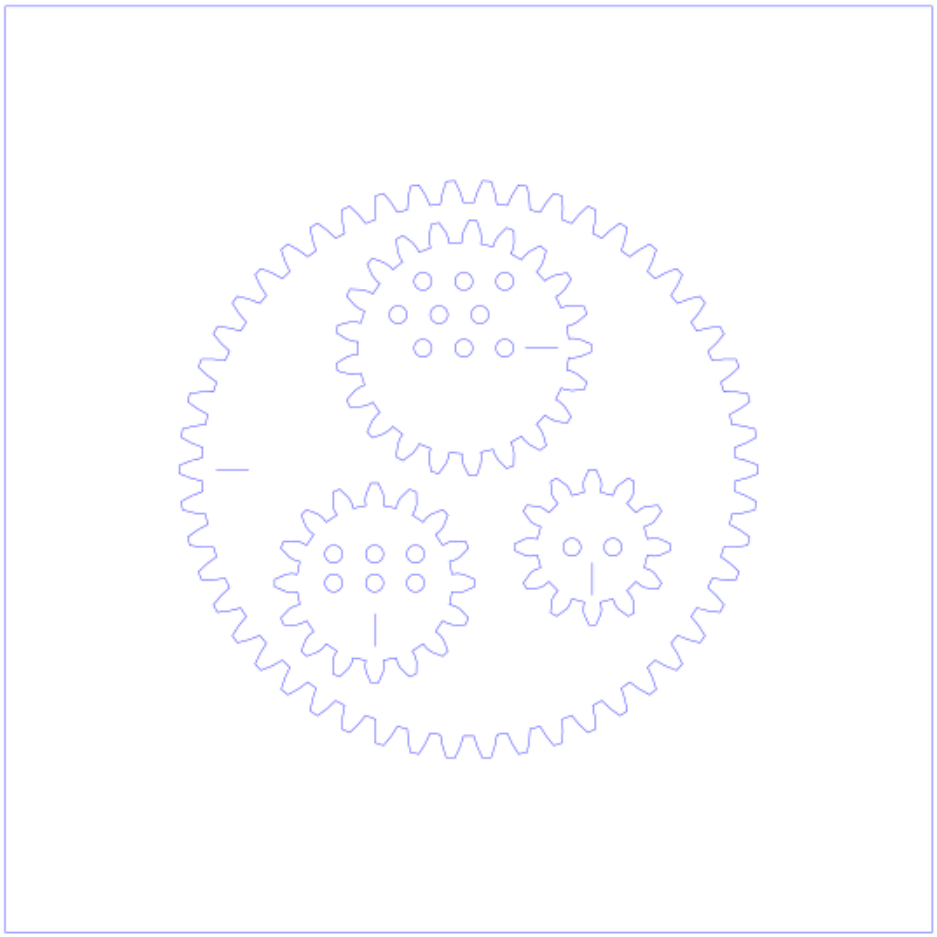
Spirograph (Wikipedia): https://en.wikipedia.org/wiki/Spirograph

Inkscape drawing software: https://inkscape.org/

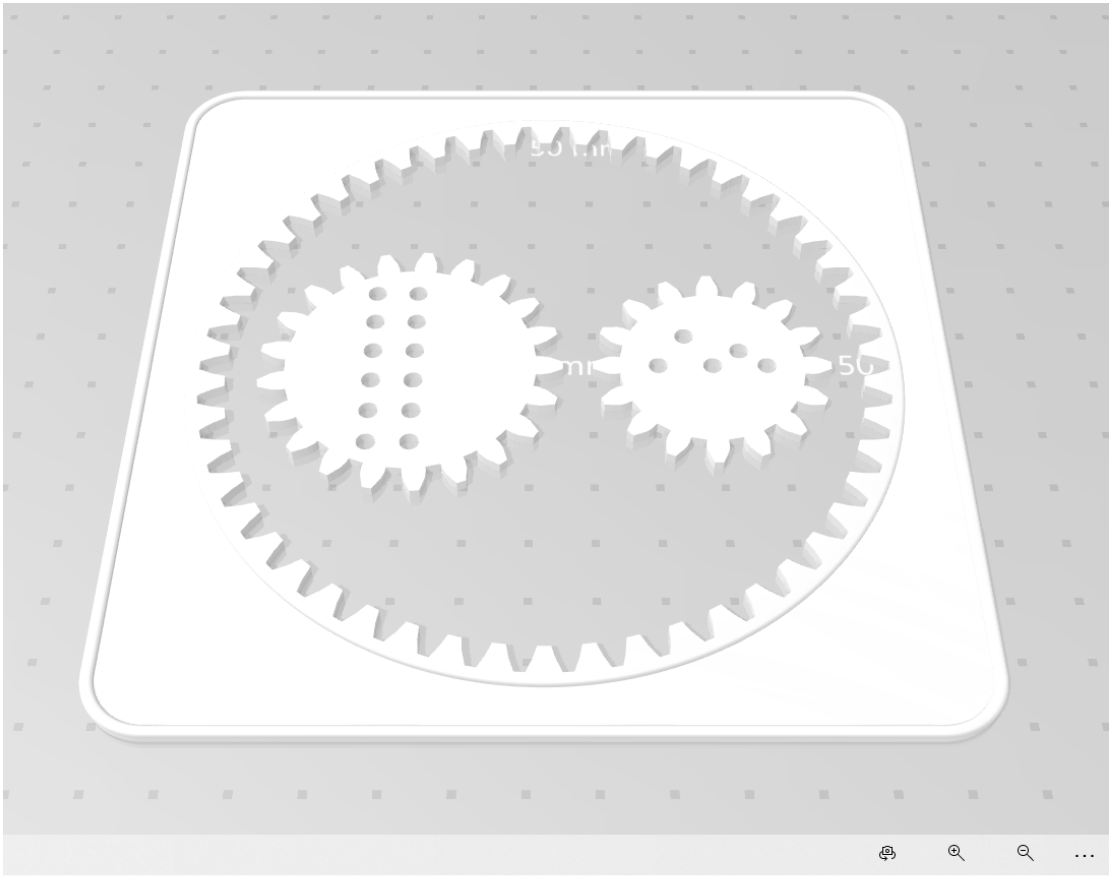
Spirograph pattern generator: https://sciencedemos.org.uk/spirograph.php

Inspirograph: https://nathanfriend.io/inspirograph/

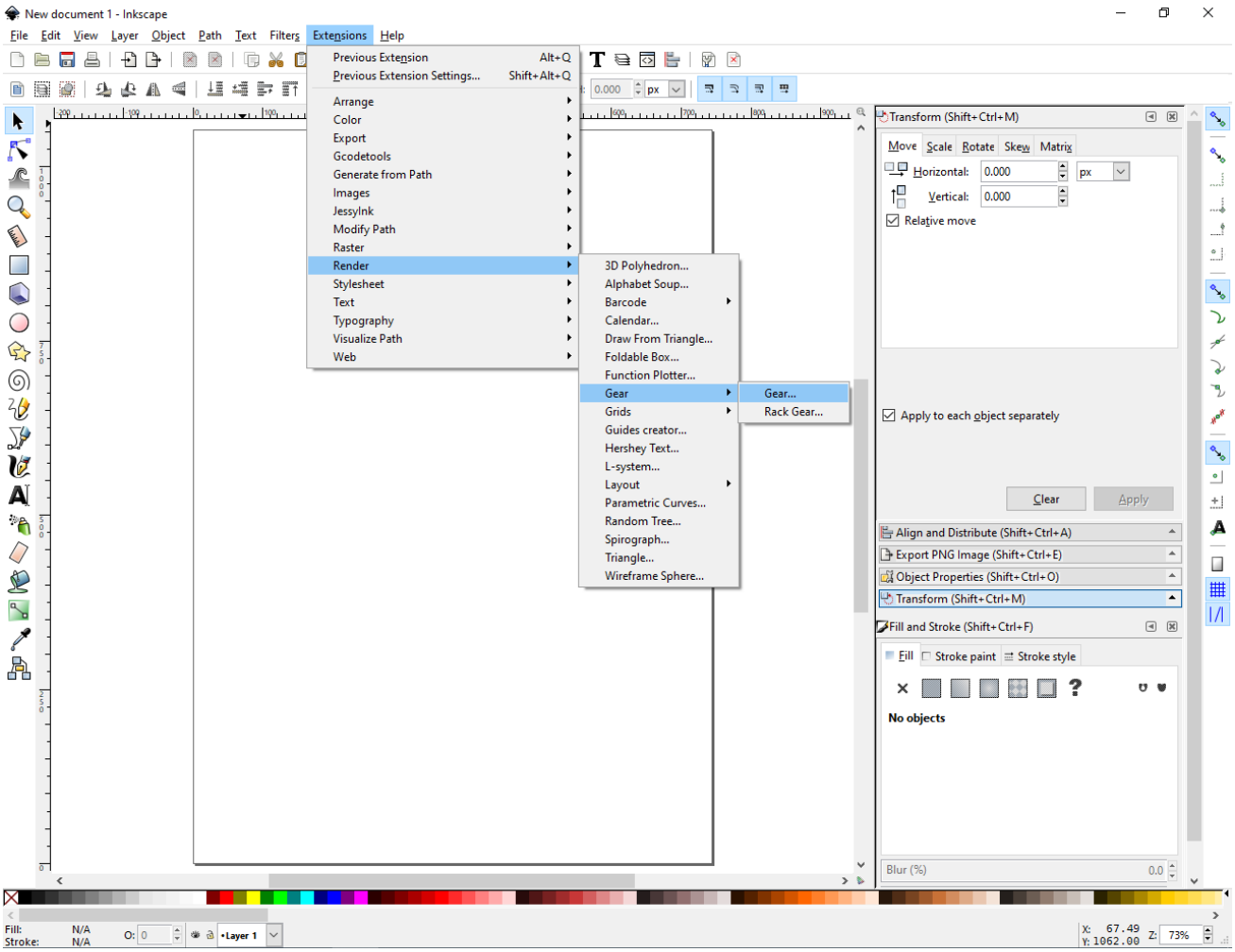
More advanced and complex cycloids: https://seedcode.com/SpirographN/sgn.html



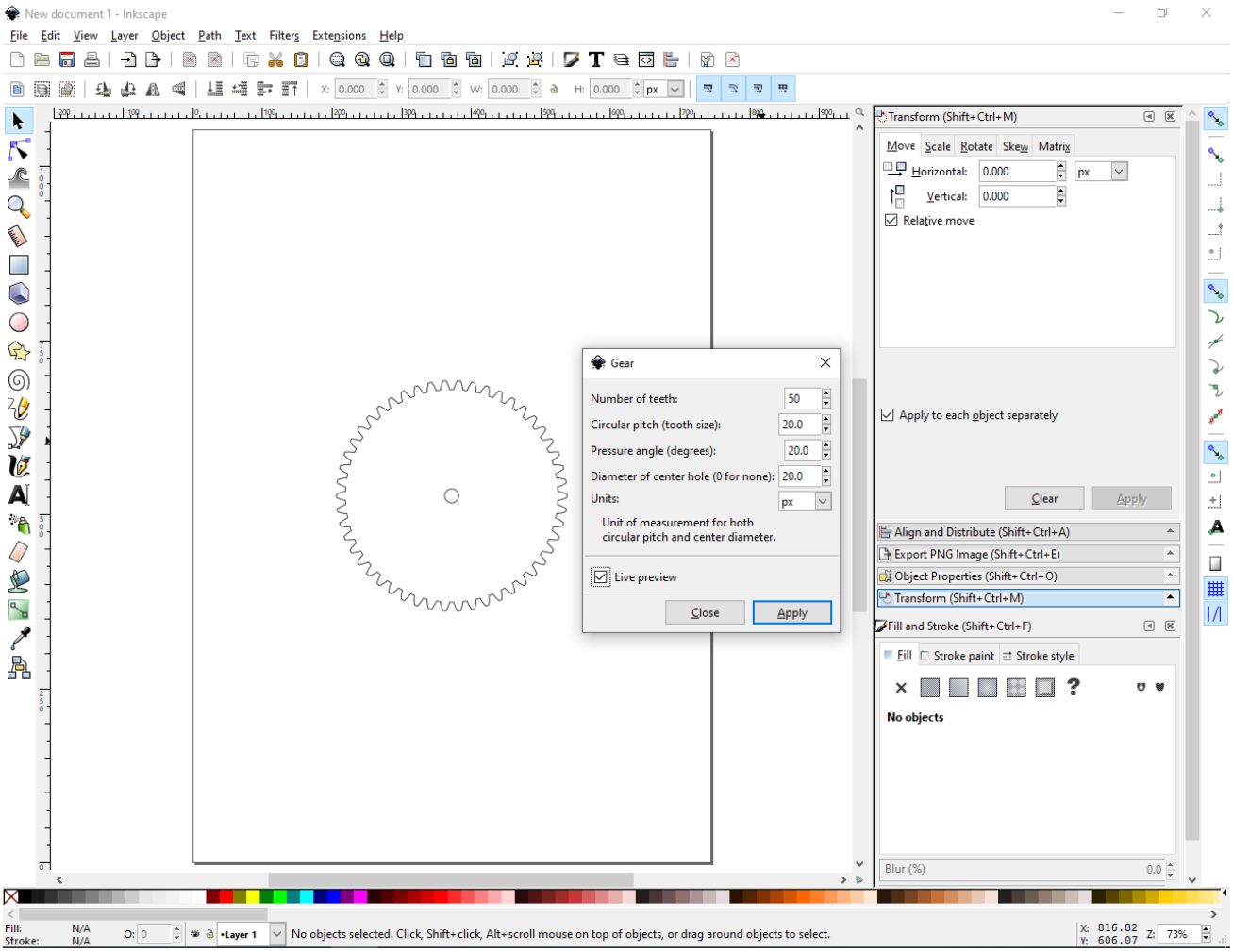
Spirograph with gears of 50, 21, 16 and 12 teeth than can be laser cut
in plexiglass or cardboard or plywood sheet



Spirograph sample that can be 3D printed in plastic



Using Inkscape to design a Spirograph. Step 1: First Go to Extensions/Render/Gear and select Gear



Using Inkscape to design a Spirograph. Step 2: The Gear window will pop-up where one can fill in various parameters of the gear to draw. The most important one is the Number of teeth. Once a large outer gear and two or more smaller gear outlines are drawn then one can make inside them small holes in various positions where the tip of the pen or pencil can go through