Maths with Spirograph

Basic Version

Georgios Mavromanolakis
[gmavroma@ea.gr]

Stimulate problem solving  Gold
Stimulate creativity  Silver
Stimulate critical thinking  Gold
Stimulate group work  Silver

Stimulate entrepreneurship  Bronze
Informal learning enviro.  Platinum
Technology use  Platinum

Practicalities

🔍 Preparation: 15min

🔎 Group size range: 15-20
Ideal sub-group size: 1 (students work individually)

⏰ Duration: 2hrs
Workshop made for: 12-16 years old
Easily transferable to workshops for ages between: -12/+16

.bluetooth Icon Environment FabLab necessary: Yes

Material needs:

● Pencils and paper sheets
● Plexiglass sheets or cardboard
● Laser cutter
● Or 3D-printer

Information Icon Educational area:
* Mathematics
* Technology
Precognition

Knowledge about arithmetic and integers. Understanding of definition of Least Common Multiple of two or more integer numbers and how to find it.

(see box ‘content links’ below)
**Preparation**

Teacher familiarize herself/himself with the operation of laser-cutter machine (preferably) or with 3D-printer that will be utilized during the workshop.

Splits and arranges students in small groups of 3-4 persons per table. Students will work individually but where necessary they share materials.

Distributes pencils or pens of various colors and paper sheets to each group.
Workshop Guidelines

Phase 1: Orientation and Instruction Phase

**Material needs**: pencils and paper sheets

*Optional: textbook of Maths or access to online educational resources*

**Goals**:

- **Skill Goals** *(Blue)*
  - (S1) Optional To look up for information independently
  - (S2) To be able to do basic arithmetic operations

- **Content Goals** *(Green)*
  - (C1) Understand and acquire content knowledge of subjects of integers and least common multiple

**Background story**:

We are in charge of a charity group that would like to distribute 150 notebooks, 90 pencils and 60 erasers in promotion packets during a fundraising campaign e.g. about climate change. How many identical packets can we make so that all contain the same number of items from the three categories?

After the completion of these tasks/activities the teacher explains students what a spirograph is and that they are going to make a real one using modern machinery. At this point she/he does not reveal how or why this is related to the maths subject they just worked on.

<table>
<thead>
<tr>
<th>Goals</th>
<th>Activities</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Ask students to look up for information about the definition of least common multiple of integer numbers and how it is calculated.</td>
<td>To be chosen</td>
</tr>
<tr>
<td>S2</td>
<td>Ask students to try to solve the teaser problem above or something similar</td>
<td>10min</td>
</tr>
<tr>
<td>C1</td>
<td>Let students to experiment with various similar problems with two or more integer numbers they pick up themselves.</td>
<td>Max: 10’</td>
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Phase 2: Design Phase

This is an optional phase. Students are given ready-to-make drawings of spirographs (see attached at the end of this document) to directly laser cut or 3D-print. One spirograph will be made per group or per table of work. At advanced level, students may use free online CAD software to draw/design their own spirograph

Material needs: Plexiglass sheets or cardboard for laser cutter. Instead of laser cutter, a 3D-printer can be used

Goals:
Skill Goals (Blue)
(S1) Abstraction of a mathematical concept to a tangible object or model

Content Goals (Green)
(C1) Experiment, identify and understand the strengths and weaknesses of different materials
(C2) Operating modern manufacturing/prototyping equipment such as laser cutter or/and 3D printer

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<tbody>
<tr>
<td>S1, C1, C2</td>
<td>Students build initial prototype versions of given spirograph designs with different materials e.g. plexiglass, paperboard, wood sheets etc. They try to understand the properties of each material and what is best for. By doing so they try to devise an optimal design, change and adapt it to achieve their goal.</td>
<td>20 min</td>
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</table>
Phase 3: Making Phase

Material needs: Plexiglass sheets or cardboard for laser cutter. Instead of laser cutter, a 3D-printer can be used

Goals:

Skill Goals (Blue)
(S1) Working under constraints (time pressure, safety regulations)

Content Goals (Green)
(C1) Experiment, identify and understand the strengths and weaknesses of different materials

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<tr>
<td>S1, C1</td>
<td>Students under close supervision of teacher use laser cutter to make the given spirograph designs with different materials e.g. plexiglass, paperboard, wood sheets etc. They follow mandatory precaution and safety measures and instructions for the operation of equipment</td>
<td>20 min</td>
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</table>
Phase 4: Operational Phase

Material needs:

**Essential:** Spirographs made in previous phase. Pencils or pens of various colors

Goals:

Skill Goals (**Blue**)  
(S1) Trial and error / Deal with failure  
(S2) Abstraction of a mathematical concept to a tangible object or model

Content Goals (**Green**)  
(C1) Practical understanding and acquisition of content knowledge of mathematical subjects of integers and least common multiple

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<td>S1, S2, C1</td>
<td>In the first subphase students are instructed to carefully count and note down the number of teeth of the gears of their spirograph at hand. They use pencils or pens of various colors to draw beautiful spirograph curves.</td>
<td>10 min</td>
</tr>
<tr>
<td>S1, S2, C1</td>
<td>They then asked if they can predict after how many revolutions the curves start again for a given set of two gears (see gears problem and drawing below). Is this related to least common multiple? They try and test their predictions. <strong>Gears problem</strong> Suppose there are two meshing gears in a machine, having $m$ and $n$ teeth, respectively, and the gears are marked by a line segment drawn from the center of the first gear to the center of the second gear. When the gears begin rotating, we can determine how many rotations the first gear must complete to realign the line segment by making use of least common multiple of $m$ and $n$ integers, usually denoted by $\text{LCM}(m, n)$. The first gear must complete $\text{LCM}(m, n)/m$ rotations for the realignment. By that time, the second gear will have made $\text{LCM}(m, n)/n$ rotations.</td>
<td>10 min</td>
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Gear with m teeth

Gear with n teeth
Phase 5: Conclusion Phase

Material needs: Multicolor spirograph drawings made in previous phase

Goals:

Skill Goals (Blue)
(S1) Abstraction of a mathematical concept to a tangible object or model
(S2) Visualization and demonstration of a mathematical concept

Content Goals (Green)
(C1) Clear understanding and acquisition of content knowledge of mathematical subjects of integers and least common multiple

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| S1, S2, C1  | Students in each table select their best multicolor spirograph drawing that demonstrates clearly the notion of least common multiple of two integer numbers. Students should be able to explain verbally their conclusions (see Gears problem in previous phase).  
Also students may be asked to reflect on the activity, for example talking about some difficulties they had, how they dealt with these problems etc. | 10 min   |
### Pedagogical tips

The workshop can be conducted in different order of phases (e.g. start with making and playing with a spirograph and then introduce the mathematical concept of least common multiple to study on) depending on the level of students. In this way students may link more easily the abstract/mathematical and tangible aspects of the learning activities in the workshop.

### How to transfer to (non-)Fablab environment

Transfer to non-fablab environment is feasible. In one case the necessary building materials are sheets of easy to cut foamboards or paperboard. Instead of gears students can draw and cut circles to make a spirograph. Close supervision and care are needed when students use hobby knifes or cutters to cut their drawings.

Alternatively, simple plastic or paper spirographs can be bought at low cost from arts and crafts or hobby shops, gadget and fun shops etc.

### Evaluation of achievements

At the end of the workshop teacher or a committee of volunteer students can award students or teams of students depending on achievements.

For example, award for:

- The most aesthetically pleasing multicolor spirograph drawing
- The most well-built spirograph
- The most quiet and efficient team of students completing all tasks
- ...

### Content links
The workshop can be enhanced with various online educational resources, interactive simulations and online generators which may be available in various languages.


Spirograph pattern generator: https://sciencedemos.org.uk/spirograph.php

Inspirograph: https://nathanfriend.io/inspirograph/

More advanced and complex cycloids: https://seedcode.com/SpirographN/sgn.html
Spirograph with gears of 50, 21, 16 and 12 teeth than can be laser cut in plexiglass or cardboard or plywood sheet
Spirograph sample that can be 3D printed in plastic